# Anisotropic thermal properties measurements by analytical Green Function solution and steady-periodic heating technique

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This paper presents both experimental and theoretical work concerning evaluation of the thermal properties of composite materials. An effective measurement system has been designed and implemented based on steady-periodic heating. The technique utilizes  $3\omega$  measurements in high and low frequency ranges separately. Measurements are carried out on a carbon-fiber epoxymatrix material.

### Experiment

A thin insulating layer of low viscosity epoxy is brushed on the surface of the sample to electrically isolate it from the metal heating lines since the carbon part of the sample is electrically conductive. A narrow, electrically conducting platinum film is deposited directly on the surface of the sample, serving simultaneously as a heater and a thermometer. The experimental setup is similar to that in Cahill [1], with the use of a SR 830 digital signal processing (DSP) lock-in amplifier with higher harmonic detection removes the need for a frequency tripling circuit. The lock-in amplifier greatly simplified the circuit since it was used for the input current, reference signal, and measurement of the third harmonic voltage. The internal oscillator built in the lockin generates a sinusoidal input current at frequency  $\omega$ passing through the platinum heater/thermometer and a regular resistor with fixed resistance, which creates a temperature fluctuation at  $2\omega$  and leads to a  $3\omega$  voltage oscillation across the sensor on the specimen. So voltage at frequencies both  $\omega$  and 3  $\omega$  with different amplitude are presented across the specimen, and the 3  $\omega$  fluctuation in the specimen embodies the information about the thermophysical properties of the material. The output voltage across the regular resistor at frequency  $\omega$  is adjusted to be close to the voltage across the sample at  $\omega$ by a 12 bit multiplying digital to analog converter (AD 7541AKNZ) whose gain can be varied from 0 to 1 by computer control of 12 TTL levels. Then the  $1\omega$ component within the  $3\omega$  signal is reduced by the differential input of the lock-in amplifier to an acceptable level. The phase lag between the  $3\omega$  component of the signal across the sample and the reference signal at  $3\omega$  (the lock-in can detect signals at harmonics of the reference frequency) is monitored by the lock-in amplifier.

### Theory

Theory for the temperature response at frequency  $2\omega$ in the specimen with the front surface heated periodically at frequency  $\omega$  has been derived analytically using a Green Function solution [2]. At high frequencies, the sample's anisotropy is negligible, and a one dimensional model is used. At low frequencies, a two dimensional model is needed. The steady periodic voltage solution at frequencies  $3\omega$  across the sample (which is the signal that lock-in amplifier actually sees) can be obtained by taking the product of the supplied current and the oscillating part of the senor resistance due to oscillating temperature.

# **Parameter Estimation**

The amplitude and phase lag between the  $3\omega$  component of the signal across the sample and the reference signal at  $3\omega$  are computed from the above solution. The parameter estimation is carried out with a least square optimization method. That is, the unknown thermal properties will be deduced from a systematic comparison between experimental data and the theoretical model.

# References

[1] D. G. Cahill, "Thermal conductivity measurement from 30 to 750K: the  $3\omega$  method", Sci. Instru. 61 (2), February 1990, pp. 802-808.

[2] K. D. Cole, "Steady-periodic Green's Functions and Thermal-measurement Applications in Rectangular Coordinates", Journal of Heat Transfer, 128 (7), 2006, pp. 709-716.